**West Coast Collaborative**

**Specialist Mathematics Unit 4 Investigation 4, 2016**

**Simple Harmonic Motion**

**Take Home Section – due Thursday 18 August**

**Validation of your findings will take place on that day, with 100% weighting on the validation.**

**Neither Calculator nor notes will be allowed in the validation.**

*Answer all questions on separate paper and provide full working to justify your answers.*

**Part A—The acceleration**

A body is said to oscillate with Simple Harmonic Motion on a straight line if the acceleration on the body is given by the differential equation:

 ,………. (1)

Where x is the distance of the body from the origin (x = 0) and the body is stationary at points.



1. Investigate the magnitude and direction of the acceleration at various points on the number line including where 
2. Hence or otherwise complete the statements:

‘The acceleration always points ………………………………………………………*’*

‘The magnitude of the acceleration is at a maximum... ……………………………*’*

‘The magnitude of the acceleration is at a minimum... …………………………….*’*

**Part B—The velocity**

We can learn about the velocity of the body by solving the differential equation (1) from above.

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| --- | --- |
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1. Complete the solution to the differential equation above to show: 
2. Investigate the magnitude and direction of the velocity at various points on the number line including where 
3. Hence or otherwise complete the statement:

‘At a given point the magnitude of the velocity is ……………. but the direction is…………….’

**Part C—The displacement**

We can learn about the position of the body by solving equation (2) from above.

We can use either the positive or negative version for the solution and we will obtain a similar result. We shall only consider the solution to the positive equation here.



1. Use the substitution given above to show the solution to the integral is:



**Part D—The phase shift**

* + To describe the displacement of the body at a particular time on the number line we need more information.
  + Usually we get this information by considering where the body is at the start, t=0

e.g. when t=0 x = 0

* + The direction of motion is also important but we ignored the velocity in equation (2)
  + Thus we need to consider the direction of the velocity,  when evaluating the constant of integration, .

1. Investigate the displacement equation (3) for the body at different starting positions and direction of motion and hence evaluate in each case.

**Part E—Change of origin**

A body is moving with Simple harmonic motion on a straight line and the acceleration is given by the equation: where x is the distance of the body from the origin x=0 and also the body is stationary at points .



1. (a) Complete the solution below to show equation for the velocity of the body is given by .



(b) Also show 

(c) Use the equation  to show 

(d) Also given that when t=0 the body is at x= +7 and the velocity is negative give the equation which shows the position x as a function time as shown in equation (3).